UNITED STATES PATENT APPLICATION

FOR

METHOD AND APPARATUS TO REMOVE EFFECTS OF I-Q IMBALANCES OF QUADRATURE MODULATORS AND DEMODULATORS IN A MULTI-CARRIER SYSTEM

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METHOD AND APPRATUS TO REMOVE EFFECTS OF I-Q IMBALANCES OF QUADRATURE MODULATORS AND DEMODULATORS IN A MULTI-CARRIER SYSTEM

CROSS-REFERENCES TO RELATED APPLICATIONS

[0001]   This application claims the benefit of U.S. Provisional Application Nos. 60/257,697 filed on December 21, 2000 (Attorney Docket No. 005281.P006Z), 60/258,111 filed on December 26, 2000 (Attorney Docket No. 005281.P007Z), and 60/262,804 filed January 19, 2001 (Attorney Docket No. 005281.P008Z).

FIELD OF THE INVENTION

[0002]   This invention relates to communication systems. In particular, the invention relates to multi-carrier systems.

BACKGROUND OF THE INVENTION

[0003]   A radio frequency (RF) communication system typically has a receiver and a transmitter. The transmitter includes a modulator to modulate a radio carrier with information or message and a power amplifier to emit the radio signal into open space. The receiver has RF stages that converts the received radio signal into some appropriate Intermediate Frequency (IF) signal and a demodulator that converts the IF signal into a baseband signal and recovers the embedded modulating information in the received signal sent by the transmitter. In a multi-carrier system, the transmitted radio signal consists of a number of signals each of which has a sub-carrier frequency and carries independent information. The sub-carrier frequencies are spaced with a fixed constant such that the sub-carrier frequencies are orthogonal each other over a certain given symbol duration. The sub-carrier frequencies are usually symmetrical around the center frequency which is the radio carrier frequency for a radio multi-carrier signal or is 0 Hz or DC for a baseband multi-carrier signal.

[0004]   Typically, the processing of the signals in the transmitter or receiver involves use of quadrature modulation or demodulation, respectively. For a quadrature

demodulator, the received signal is down-converted into a complex baseband signal by in-phase (I) and quadrature (Q) mixers and the baseband signal is split into in-phase (I) and quadrature (Q) components to be processed separately in the respective I and Q channels. These two components are then usually filtered and converted into digital samples. For a quadrature modulator, a reverse procedure is carried out. In real systems, there are a number of problems associated with the processing of the signals, which result in poor performance. These problems include mismatch or imbalance in gain or group delays between the I and Q channels. As a result, the sub-carrier signals in a multi-carrier system may not completely satisfy the orthogonality over the symbol duration. These problems introduce cross-talks between the sub-carriers and distortion in the received or transmitted signals.

[0005]   Therefore, there is a need to have an efficient technique to remove the adverse effects of the I-Q imbalances in a multi-carrier system.

BRIEF DESCRIPTION OF THE DRAWINGS

[0006]   The features and advantages of the present invention will become apparent from the following detailed description of the present invention in which:

[0007]   Figure 1 is a diagram illustrating a system in which one embodiment of the invention can be practiced.

[0008]   Figure 2 is a diagram illustrating a balanced demodulator unit shown in Figure 1 according to one embodiment of the invention.

[0009]   Figure 3 is a diagram illustrating a balanced modulator unit shown in Figure 1 according to one embodiment of the invention.

[0010]   Figure 4 is a diagram illustrating the in-phase and quadrature non-orthogonality according to one embodiment of the invention.

[0011]   Figure 5 is a diagram illustrating a basic balancing block shown in Figure 1 according to one embodiment of the invention.

DETAILED DESCRIPTION OF THE INVENTION

[0012]   In the following description, for purposes of explanation, numerous details are set forth in order to provide a thorough understanding of the present invention. However, it will be apparent to one skilled in the art that these specific details are not required in order to practice the present invention. In other instances, well-known electrical structures and circuits are shown in block diagram form in order not to obscure the present invention. It is also noted that the invention may be described as a process, which is usually depicted as a flowchart, a flow diagram, a structure diagram, or a block diagram. Although a flowchart may describe the operations as a sequential process, many of the operations can be performed in parallel or concurrently. In addition, the order of the operations may be re-arranged. A process is terminated when its operations are completed. A process may correspond to a method, a function, a procedure, a subroutine, a subprogram, etc. When a process corresponds to a function, its termination corresponds to a return of the function to the calling function or the main function.

[0013]   Figure 1 is a diagram illustrating a system 100 in which one embodiment of the invention can be practiced. The system 100 may be a wireless communication system or any communication system with similar characteristics. The system 100 includes an antenna 105, a receive/transmit switch 110, a receiver 120, a transmitter 125, a local oscillator 130, and a self-calibration switch 135. Not all of the elements are required for the system 100.

[0014]   The antenna 105 receives and transmits radio frequency signal. In one embodiment, the received and transmitted signals are in a frequency range of 5.2 GHz – 5.7 GHz. The received and transmitted signals are multi-carrier signals having a number of sub-carriers. Each of the multi-carrier signals is a composite signal consisting of sub-carrier signals at a number of sub-carrier frequencies. The sub-carriers are separated by a fixed frequency separation. The receive/ transmit switch 110 connects the antenna to the receiver 120 or the transmitter 125 depending on whether the system 100 is in the receive mode or transmit mode, respectively. When the system 100 is configured as either a receiver or a transmitter, the receive/transmit switch 110 is not needed. The local oscillator

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130 generates oscillating signal at an appropriate frequency to down convert the received signal to baseband for the receiver 120, or to up convert the baseband signal to appropriate transmission frequency for the transmitter 125. The self-calibration switch 135 is used for self-calibration or training. The self-calibration switch 135 is open in normal operation when the receiver 120 and the transmitter 125 operate independently. The self-calibration switch 135 is closed to allow locally generated signals to be processed by the receiver 120 during training. Alternatively, the self-calibration switch 135 is not needed and the training can be performed with a remote transmitter.

[0015] The receiver 120 processes the received signal, which carries the multi-carrier baseband signal $Y(t) = \sum_{k=-N}^{N} X(k) \cdot \exp(j2\pi k\Delta_F t)$ plus some noise and distortation if any and recovers the original modulating signals $X(-N)$, . . , $X(-k)$, . . ., $X(k)$, . . . , and $X(N)$, indexed by integers $-N$, ...-k, ...k, ...,N, where the k-th signal corresponds to the original signal from the k-th sub-carrier modulator whose sub-carrier frequency is $k\Delta_F$ higher than the center frequency of the multi-carrier signal (while index $-k$ means $k\Delta_F$ lower than the center frequency). The index 0 corresponds to a center frequency of the composite multi-carrier signal which is 0 Hz for the complex baseband signal and can be any radio frequency if the baseband signal is up-converted to any radio frequency band. The receiver includes a low noise amplifier (LNA) 140, a mixer 145, a band-pass filter (BFP) 150, an amplifier/ buffer 155, and a balancing demodulator unit 160. The LNA 140 amplifies the received signal with low noise. The mixer 145 acts like an analog multiplier to multiply the received signal with the oscillating signal from the local oscillator 130. The band pass filter 150 filters the multiplied signal to allow signal at the desired pass band centered at an intermediate frequency (IF) ω to pass through. The amplifier 155 amplifies the band-passed signal with appropriate gain for further processing. The balancing demodulator unit 160 performs the final demodulation to separate the composite base-band signal into the original signals $X(-N)$, . . , $X(-k)$, . . ., $X(k)$, . . . , and $X(N)$ at appropriate sub carrier frequencies

[0016]   The transmitter 125 processes the mutually independent original signals *X(-N)*,
. . ., *X(-k)*, . . ., *X(k)*, . . ., and *X(N)* indexed by -*N*, ...,-*k*, ...,*k*, ...,*N*, and generates a
composite signal for transmission. The transmitter 125 includes a balanced modulator unit
165, a filter 170, an amplifier 175, a mixer 180, a band-pass filter (BPF) 185, and a power
amplifier 190. The balanced modulator unit 165 receives the original signals at suitable
sub-carrier frequencies. It generates a composite signal to the filter 170. The filter 170
eliminates unwanted spectral components. The amplifier 175 amplifies the signal with
suitable gain. The mixer 180 acts like an analog multiplier to multiply the signal with the
local oscillator 130. The band pass filter 185 allows the signal at the frequency band of
interest to pass through. The PA 190 amplifies the band-passed signal with proper power.
The final signal goes to the receive/transmit switch 110 to be transmitted.

[0017]   Figure 2 is a diagram illustrating the balancing demodulator unit 160 shown in
Figure 1 according to one embodiment of the invention. The balancing demodulator unit
160 includes a local oscillator 203, a phase splitter 205, an in-phase (I) mixer 210, an in-
phase low-pass filter (LPF) 215, an in-phase analog-to-digital converter (ADC) 220, a
quadrature (Q) mixer 230, a quadrature LPF 235, a quadrature ADC 240, a bank of sub-
carrier demodulators 250, and a demodulator balancing block 260.

[0018]   The phase splitter 205 generates cosine and sine waveforms at the intermediate
frequency ω provided by the local oscillator 203. The cosine and sine waveforms are
ideally orthogonal. As will be explained later, if they are not orthogonal, the resulting non-
balanced demodulated signals may have distortion. The I and Q mixers 210 and 230 act
like analog multipliers to multiply the input signal with the $\cos(\omega t)$ and $-\sin(\omega t)$ to
essentially convert the input signal to the base-band frequency. The I and Q LPFs 215 and
235 filter out the high frequency spectral components of the corresponding mixed signal to
allow the signal at base-band to pass through. The I and Q ADCs 220 and 240 convert the
base-band signal to digital samples.

[0019]   However, due to variations in manufacturing process for analog components
such as mixers, filters, and ADCs/DACs, the overall frequency/group delay and gain
response of the I channel may be different from that of the Q channel and the local

reference signals to the I-Q mixers may not be orthogonal (90 degrees in phase). They are called I-Q mismatch or imbalance. Therefore, in Figure 2, the complex signal at the output the I-Q channels $\hat{Y}(t)$ is a distorted version of $Y(t)$. Similarly for the I-Q modulator in Figure 3, if we were to apply $\tilde{Y}(t)$ to the input of I-Q channels of the I-Q modulator, the resulted signal is the real part of $Y(t)\exp(j\omega t)$ where $Y(t)$ may be a distorted version of $\tilde{Y}(t)$ due to I-Q imbalances.

[0020]   The bank of sub-carrier demodulators 250 includes $2N+1$ demodulators operating at sub-carrier frequencies separated by a fixed frequency separation Δ_F. The bank 250 receive the digital I and Q samples and perform sub-carrier demodulation to generate $2N+1$ non-balanced signals $\hat{X}(-N),...,\hat{X}(-k),...,\hat{X}(k),...,\hat{X}(N)$. The k-th sub-carrier demodulator performs the function $\hat{X}(k) = \dfrac{1}{T}\int_0^T \hat{Y}(t)\cdot\exp(-j2\pi k\Delta_F t)dt$, where T is the symbol duration for the input signal $\hat{Y}(t)$. Due to noise, imbalance in gain, and non-orthogonality of the mixers 210 and 230, the non-balanced signals may be distorted or exhibit cross-talk noise. The demodulator balancing block 260 compensates for the distortion or noise by balancing these effects. The demodulator balancing block 260 recovers the original signals and generates the original signals $X(-N),...,X(-k),...,X(k),...,X(N)$. The demodulator balancing block 260 includes $N+1$ basic blocks 265_0, 265_1 to 265_N. The basic block 265_0 processes $\hat{X}(0)$. Each of the N basic blocks from 265_1 to 265_N processes a pair of non-balanced signals from the sub-carrier demodulators with indices symmetrical around index 0 . In other words, each of the N basic blocks processes $\hat{X}(k)$ and $\hat{X}(-k)$ and generates $X(k)$ and $X(-k)$ up to differences of constant complex scales which can be removed by referencing to known pilot or training signals.

[0021]   As is known by one skilled in the art, the techniques of the present invention are applicable to any other types of signal decomposition. The signal Y(t) may be represented by any method, including representation in the Fourier, Hilbert, and analytic domains. The sub-carrier signals may be components of the composite signal Y(t) as Fourier Transform elements, Fourier series expansion elements, complex envelope

components, etc. Furthermore, the techniques may also be applicable or extended to signal crosstalks or distortions caused by reasons other than I-Q imbalances.

[0022]   Figure 3 is a diagram illustrating the balancing modulator unit 165 shown in Figure 1 according to one embodiment of the invention. The balancing modulator unit 165 includes a modulator balancing block 310, a bank of sub-carrier modulators 320, I and Q digital-to-analog converters (DACs) 330 and 360, I and Q LPF 335 and 365, I and Q mixers 340 and 370, a phase splitter 350, a local oscillator 353, and a combiner 380.

[0023]   The modulator balancing block 310 processes $2N+1$ original modulating signals $X(-N), \ldots, X(-k), \ldots, X(k), \ldots, X(k), \ldots, X(N)$. The modulator balancing block 310 generates $2N+1$ pre-compensated signals $\widetilde{X}(-N), \ldots, \widetilde{X}(-k), \ldots, \widetilde{X}(k), \ldots, \widetilde{X}(N)$ to the bank of sub-carrier modulators 320 by compensating or balancing the effects due to imbalances in gain mismatch in the I and Q LPFs 335 and 365, phase non-orthogonality in the mixers 340 and 370. The modulator balancing block 310 includes $N+1$ basic blocks 315_0, 315_1 to 315_N. The basic block 315_0 processes $X(0)$. Each of the basic blocks 315_1 to 315_N processes a pair of signals $X(-k)$ and $X(k)$ having symmetrical indices around index 0 . The bank of sub-carrier modulators 320 includes a number of complex sub-carrier modulators and combines the outputs of all sub-carrier modulators. A sub-carrier modulator at frequency $k\Delta_F$ generates $\widetilde{X}(k) \cdot \exp(j2\pi k\Delta_F t)$ over a given symbol duration T. Therefore, the bank of sub-carrier modulators 320 generates a multi-carrier baseband signal $\widetilde{Y}(t) = \displaystyle\sum_{k=-N}^{N} \widetilde{X}(k) \cdot \exp(j2\pi k \Delta_F t)$. The real part and the imaginary part of $\widetilde{Y}(t)$ become the I and Q components of the composite signal for the following DACs , low-pass filters and the I-Q modulator to be up-converted to a radio signal with a center angle frequency ω, respectively.

[0024]   The I and Q DACs 330 and 360 convert the I and Q digital samples from the bank of sub-carrier modulators 320 into analog signals. The I and Q LPFs 335 and 365 filter the corresponding I and Q signals from the I and Q DACs 330 and 360. Any imbalance in gain in the I and Q LPFs 335 and 365 is balanced by the modulator balancing block 310. The I and Q mixers 340 and 370 mix the corresponding I and Q signals with

the cos(ωt) and −sin(ωt) provided by the phase splitter 350 to generate signals at appropriate frequencies. The local oscillator 353 provides the frequency source to the phase splitter 350. The combiner 380 combines the I and Q signals into a composite signal to be transmitted.

[0025] Figure 4 is a diagram illustrating the in-phase and quadrature non-orthogonality according to one embodiment of the invention. It is noted that when this happens, the baseband component of the output of the I-Q mixers of the demodulator in Figure 2 is no longer Y(t) and neither is the complex envelope of the output of the I-Q mixers of the modulator in Figure 3.

[0026] As shown, in the ideal situation, the phase difference between the phase splitter used in the mixing of the I and Q channels is 90 degrees, such that the cos(ωt) and -sin(ωt) components are orthogonal. In practice, there is some amount of non-orthogonality caused by a phase deviation φ. The cos(ωt) and -sin(ωt) components become cos(ωt-φ) and -sin(ωt+φ), respectively. This phase deviation may cause signal distortion. The balancing blocks used in the modulator and demodulator also compensate for this phase deviation.

[0027] The balancing blocks are implemented based on derivation of the balancing parameters used in the balancing blocks and a training process to determine these parameters. The derivation is explained below.

[0028] <u>Demodulator:</u>

[0029] Refer to Figure 2 showing a demodulator unit. Let ω be the radio carrier frequency and Δ_F be the frequency spacing between sub-carriers. Further, suppose that the sub-carriers satisfy the orthogonality over symbol duration T: i.e.,

$$\frac{1}{T}\int_0^T \exp(j2\pi m\Delta_F t)\exp(-j2\pi n\Delta_F t)dt = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases} \tag{1}$$

[0030] The input signal of the demodulator is $\text{Re}\left[Y(t)\cdot\exp(j\omega\cdot t)\right]$ where Re[.] is the real part of complex variable and $Y(t) = \sum_{k=-N}^{N} X(k)\cdot\exp(j2\pi k\Delta_F t)$ is the modulated multi-

carrier low-pass equivalent signal with $\{X(k)$ where $k=-N,...,N,$ k is an integer$\}$ modulating the $2N+1$ sub-carriers, respectively.

[0031] Suppose that in Figure 2 the low-pass filters 215 and 235 have filter characteristics, or transfer functions, $H_I(\omega)$ and $H_Q(\omega)$, respectively. The effects of the ADCs 220 and 240 and other analog components along the I-channel or Q-channel may be included in the low-pass filters 215 and 235. Suppose the I and Q channels have gain and group delay profiles $\{G_I(k),\tau_I(k):k=0,...,N\}$ and $\{G_Q(k),\tau_Q(k):k=0,...,N\}$, respectively. $G_I(k)$ and $\tau_I(k)$ are the gain and group delay of the I-channel at frequency $k\Delta_F$. $G_Q(k)$ and $\tau_Q(k)$ are the Q-channel gain and group delay at frequency $k\Delta_F$. Assume that $\tau_I(k)$, $\tau_Q(k)$, $G_I(k)$, and $G_Q(k)$ are almost constant over the frequency bandwidth Δ_F centered at frequency $k\Delta_F$. Define $\theta_I(k)=2\pi\cdot k\Delta_F\tau_I(k)+\theta_I(0)$ and $\theta_Q(k)=2\pi\cdot k\Delta_F\tau_Q(k)+\theta_Q(0)$ as the corresponding phase shifts at frequency $k\Delta_F$ due to the group delays of the LPFs 215 and 235, where $\theta_I(0)$ and $\theta_Q(0)$ are the phase shifts due to some fixed delay at frequency 0 (e.g., delay due to ADC and DAC processes).

[0032] Consider a situation with two sub-carriers. Let the base-band signal be

$$Y(t)=X(k)\cdot \exp(j2\pi k\Delta_F t)+X(-k)\cdot \exp(-j2\pi k\Delta_F t) \tag{2}$$

for some k>0, where $X(k)=A+jB$ and $X(-k)=C+jD$. For simplicity, define $\omega_k=2\pi\cdot k\Delta_F$, $G_I=G_I(k)$ and $G_Q=G_Q(k)$, $\theta_I=\theta_I(k)$ and $\theta_Q=\theta_I(k)$. Then, the signal after the ADCs 220 and 240 can be expressed as:

$$\hat{Y}(t) = G_I \cdot \text{Re}[X(k) \cdot \exp(j(\omega_k t - \theta_I)) + X(-k) \cdot \exp(-j(\omega_k t - \theta_I))] +$$
$$+ jG_Q \cdot \text{Im}[X(k) \cdot \exp(j(\omega_k t - \theta_Q)) + X(-k) \cdot \exp(-j(\omega_k t - \theta_Q))] =$$
$$= G_I[A\cos(\omega_k t - \theta_I) - B\sin(\omega_k t - \theta_I)] +$$
$$+ jG_Q[B\cos(\omega_k t - \theta_Q) + A\sin(\omega_k t - \theta_Q)] +$$
$$+ G_I[C\cos(\omega_k t - \theta_I) + D\sin(\omega_k t - \theta_I)] +$$
$$+ jG_Q[D\cos(\omega_k t - \theta_Q) - C\sin(\omega_k t - \theta_Q)] =$$
$$= G_I[A\cos(\omega_k t)\cos\theta_I + A\sin(\omega_k t)\sin\theta_I - B\sin(\omega_k t)\cos\theta_I + B\cos(\omega_k t)\sin\theta_I] +$$
$$+ jG_Q[B\cos(\omega_k t)\cos\theta_Q + B\sin(\omega_k t)\sin\theta_Q + A\sin(\omega_k t)\cos\theta_Q - A\cos(\omega_k t)\sin\theta_Q] +$$
$$+ G_I[C\cos(\omega_k t)\cos\theta_I + C\sin(\omega_k t)\sin\theta_I + D\sin(\omega_k t)\cos\theta_I - D\cos(\omega_k t)\sin\theta_I] +$$
$$+ jG_Q[D\cos(\omega_k t)\cos\theta_Q + D\sin(\omega_k t)\sin\theta_Q - C\sin(\omega_k t)\cos\theta_Q + C\cos(\omega_k t)\sin\theta_Q] =$$
$$= G_I\cos\theta_I[A\cos(\omega_k t) - B\sin(\omega_k t)] + G_I\sin\theta_I[A\sin(\omega_k t) + B\cos(\omega_k t)] +$$
$$+ jG_Q\cos\theta_Q[B\cos(\omega_k t) + A\sin(\omega_k t)] + jG_Q\sin\theta_Q[B\sin(\omega_k t) - A\cos(\omega_k t)] +$$
$$+ G_I\cos\theta_I[C\cos(\omega_k t) + D\sin(\omega_k t)] + G_I\sin\theta_I[C\sin(\omega_k t) - D\cos(\omega_k t)] +$$
$$+ jG_Q\cos\theta_Q[D\cos(\omega_k t) - C\sin(\omega_k t)] + jG_Q\sin\theta_Q[D\sin(\omega_k t) + C\cos(\omega_k t)]$$

$$(3)$$

where Re[.] is the real part of complex variable and Im[.] is the imaginary part of complex variable

[0033]   For any $k>0$, $\hat{X}(k)$ and $\hat{X}(-k)$ are the outputs of the corresponding sub-carrier demodulators and are calculated in the following,

$$\hat{X}(k) = \frac{1}{T}\int_0^T \hat{Y}(t)(\cos(\omega_k t) - j\sin(\omega_k t))dt =$$
$$= G_I\cos\theta_I[A + jB]/2 + G_I\sin\theta_I[-jA + B]/2 +$$
$$+ jG_Q\cos\theta_Q[B - jA]/2 + jG_Q\sin\theta_Q[-jB - A]/2 +$$
$$+ G_I\cos\theta_I[C - jD]/2 + G_I\sin\theta_I[-jC - D]/2 +$$
$$+ jG_Q\cos\theta_Q[D + jC]/2 + jG_Q\sin\theta_Q[-jD + C]/2 =$$
$$= G_I\cos\theta_I[A + jB]/2 - jG_I\sin\theta_I[A + jB]/2 +$$
$$+ G_Q\cos\theta_Q[A + jB]/2 - jG_Q\sin\theta_Q[A + jB]/2 +$$
$$+ G_I\cos\theta_I[C - jD]/2 - jG_I\sin\theta_I[C - jD]/2 +$$
$$- G_Q\cos\theta_Q[C - jD]/2 + jG_Q\sin\theta_Q[C - jD]/2 =$$
$$= [(G_I\cos\theta_I + G_Q\cos\theta_Q) - j(G_I\sin\theta_I + G_Q\sin\theta_Q)](A + jB)/2 +$$
$$+ [(G_I\cos\theta_I - G_Q\cos\theta_Q) - j(G_I\sin\theta_I - G_Q\sin\theta_Q)](C - jD)/2$$

$$(4)$$

and

$$\hat{X}(-k) = \frac{1}{T}\int_0^T \hat{Y}(t)(\cos(\omega_k t) + j\sin(\omega_k t))dt =$$

$$= G_I \cos\theta_I[A - jB]/2 + G_I \sin\theta_I[jA + B]/2 +$$
$$+ jG_Q \cos\theta_Q[B + jA]/2 + jG_Q \sin\theta_Q[jB - A]/2 +$$
$$+ G_I \cos\theta_I[C + jD]/2 + G_I \sin\theta_I[jC - D]/2 +$$
$$+ jG_Q \cos\theta_Q[D - jC]/2 + jG_Q \sin\theta_Q[jD + C]/2 =$$
$$= G_I \cos\theta_I[A - jB]/2 + jG_I \sin\theta_I[A - jB]/2 + \qquad (5)$$
$$- G_Q \cos\theta_Q[A - jB]/2 - jG_Q \sin\theta_Q[A - jB]/2 +$$
$$+ G_I \cos\theta_I[C + jD]/2 + jG_I \sin\theta_I[C + jD]/2 +$$
$$+ G_Q \cos\theta_Q[C + jD]/2 + jG_Q \sin\theta_Q[C + jD]/2 =$$
$$= [(G_I \cos\theta_I + G_Q \cos\theta_Q) + j(G_I \sin\theta_I + G_Q \sin\theta_Q)](C + jB)/2 +$$
$$+ [(G_I \cos\theta_I - G_Q \cos\theta_Q) + j(G_I \sin\theta_I - G_Q \sin\theta_Q)](A - jB)/2$$

[0034]   Define $G_c(k) = (G_I \cos\theta_I + G_Q \cos\theta_Q)/2$,

$G_s(k) = (G_I \sin\theta_I + G_Q \sin\theta_Q)/2$, $\Delta_c(k) = (G_I \cos\theta_I - G_Q \cos\theta_Q)/2$, and

$\Delta_s(k) = (G_I \sin\theta_I - G_Q \sin\theta_Q)/2$. Then

$$\hat{X}(k) = (G_c(k) - jG_s(k))(A + jB) + (\Delta_c(k) - j\Delta_s(k))(C - jD) \qquad (6)$$

and

$$\hat{X}(-k) = (G_c(k) + jG_s(k))(C + jD) + (\Delta_c(k) + j\Delta_s(k))(A - jB) \qquad (7)$$

[0035]   For $k=0$ (i.e., one sub-carrier at 0 frequency) and $X(0) = A + jB$, there are

$$\hat{Y}(t) = G_I \cdot \mathrm{Re}[X(0) \cdot \exp(j(-\theta_I))] + jG_Q \cdot \mathrm{Im}[X(0) \cdot \exp(j(-\theta_Q))] =$$
$$= G_I[A\cos(\theta_I) + B\sin(\theta_I)] + jG_Q[B\cos(\theta_Q) - A\sin(\theta_Q)] \qquad (8)$$

and

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$$\hat{X}(0) = \frac{1}{T} \int_0^T \hat{Y}(t)dt =$$

$$= G_I[A\cos\theta_I + B\sin\theta_I] + jG_Q[B\cos\theta_Q - jA\sin\theta_Q] =$$

$$= [G_I\cos\theta_I - jG_Q\sin\theta_Q]A + [G_I\sin\theta_Q + jG_Q\cos\theta_Q]B =$$

$$= [G_I\cos\theta_I - jG_Q\sin\theta_Q](X(0) + X^*(0))/2 \qquad (9)$$

$$+ [G_I\sin\theta_I + jG_Q\cos\theta_Q](X(0) - X^*(0))/2j$$

$$= [(G_I\cos\theta_I + G_Q\cos\theta_Q) - j(G_I\sin\theta_I + G_Q\sin\theta_Q)]X(0)/2 +$$

$$+ [(G_I\cos\theta_I - G_Q\cos\theta_Q) + j(G_I\sin\theta_I - G_Q\sin\theta_Q)]X^*(0)/2 =$$

$$= (G_c(0) - jG_s(0))X(0) + (\Delta_c(0) + j\Delta_s(0))X^*(0)$$

[0036]   Further, define $G_{cs}(k) = G_c(k) + jG_s(k)$ and $\Delta_{cs}(k) = (\Delta_c(k) + j\Delta_s(k))$ and recall $X(k) = A + jB$ and $X(-k) = C + jD$ for $k>0$. There are

$$\hat{X}(k) = G_{cs}^*(k)X(k) + \Delta_{cs}^*(k)X^*(-k) \qquad (10)$$

and

$$\hat{X}(-k) = G_{cs}(k)X(-k) + \Delta_{cs}(k)X^*(k) \qquad (11)$$

where $(\cdot)^*$ takes the conjugate of the complex variables.

[0037]   In general, suppose that there are two sets of balancing parameters $\{G_{cs}(k) : k = 0,...,N\}$ and $\{\Delta_{cs}(k) : k = 0,...,N\}$, where

$$G_{cs}(k) = G_c(k) + jG_s(k) \text{ and } \Delta_{cs}(k) = (\Delta_c(k) + j\Delta_s(k)) \qquad (12)$$

and

$$G_c(k) = (G_I(k)\cos\theta_I(k) + G_Q(k)\cos\theta_Q(k))/2, \qquad (13)$$

$$G_s(k) = (G_I(k)\sin\theta_I(k) + G_Q(k)\sin\theta_Q(k))/2, \qquad (14)$$

$$\Delta_c(k) = (G_I(k)\cos\theta_I(k) - G_Q(k)\cos\theta_Q(k))/2, \qquad (15)$$

and

$$\Delta_s(k) = (G_I(k)\sin\theta_I(k) - G_Q(k)\sin\theta_Q(k))/2 \qquad (16)$$

and $\theta_I(k) = 2\pi \cdot k\Delta_F\tau_I(k)$ and $\theta_Q(k) = 2\pi \cdot k\Delta_F\tau_Q(k)$. Then for any $k>0$, the outputs of the bank of sub-carrier demodulator 250 are

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$$\hat{X}(k) = G_{cs}^*(k)X(k) + \Delta_{cs}^*(k)X^*(-k), \tag{17}$$

$$\hat{X}(-k) = G_{cs}(k)X(-k) + \Delta_{cs}(k)X^*(k), \tag{18}$$

[0038] From the above:

$$\hat{X}(k)G_{cs}^*(k) - (\Delta_{cs}\hat{X}(-k))^* = [(G_{cs}^*(k))^2 - (\Delta_{cs}^*(k))^2]X(k), \tag{19}$$

$$\hat{X}(-k)G_{cs}(k) - (\Delta_{cs}^*(k)\hat{X}(k))^* = [(G_{cs}(k))^2 - (\Delta_{cs}(k))^2]X(-k), \tag{20}$$

[0039] Once the parameters $G_{cs}(k)$ and $\Delta_{cs}(k)$ are known, $\{X(k) : k = -N,...,N\}$ can be recovered from $\{\hat{X}(k) : k = -N,...,N\}$ with the above operations that are the functions of the demodulator balancing block 260 in Figure 2.

[0040] Refer to the modulator unit 165 shown in Figure 3. The balancing modulator unit 165 generates $\mathrm{Re}[Y(t)\cdot\exp(j\omega\cdot t)]$ where $Y(t) = \sum_{k=-N}^{N} X(k)\cdot\exp(j2\pi k\Delta_F t)$ and {*X(k)*: *k=-N, ...,N*} is a set of symbols modulating the *2N+1* sub-carriers, respectively. However, since the I and Q channels of the modulator may be mis-matched, it may be desirable to generate $\tilde{Y}(t) = \sum_{k=-N}^{N} \tilde{X}(k)\exp(j2\pi k\Delta_F t)$ at the input of the Digital-to-Analog Converters 330 and 360 of the I and Q channels so that at the outputs of the LPFs 335 and 365 of I and Q channels will be $Y(t) = \sum_{k=-N}^{N} X(k)\cdot\exp(j2\pi k\Delta_F t)$. According to equation (17) and (18), there are

$$X(k) = G_{cs}^*(k)\tilde{X}(k) + \Delta_{cs}^*(k)\tilde{X}^*(-k), \tag{21}$$

$$X(-k) = G_{cs}(k)\tilde{X}(-k) + \Delta_{cs}(k)\tilde{X}^*(k), \tag{22}$$

[0041] Therefore,

$$X(k)G_{cs}^*(k) - (\Delta_{cs}X(-k))^* = [(G_{cs}^*(k))^2 - (\Delta_{cs}^*(k))^2]\tilde{X}(k), \tag{23}$$

$$X(-k)G_{cs}(k) - (\Delta_{cs}^*(k)X(k))^* = [(G_{cs}(k))^2 - (\Delta_{cs}(k))^2]\tilde{X}(-k), \tag{24}$$

[0042] From the above equations, given the desired modulating symbols *{X(k): k=-N,...,N}*, $\widetilde{X}(k)$ and $\widetilde{X}(-k)$ may be generated for the corresponding sub-carrier modulators by introducing "cross-talks" between *X(k)* and *X(-k)*.

[0043] It is next to derive how to obtain the balancing parameters for the modulator and demodulator balancing blocks 260 (Figure 2) and 310 (Figure 3). There are two ways to do this. One is to train locally and one is to train with remote units. The local self training or self calibration technique for the demodulator block 260 uses the modulator in a system that has both modulator and demodulator.

[0044] The balancing modulator unit 165 is used to send out ideal signals to the balancing demodulator unit 160 by closing the self-calibration switch 135 (Figure 1). The ideal signal can be used as a reference signal to obtain the balancing parameters for the balancing demodulator unit 160. To distinguish the parameters of modulator and demodulator, define $\{G_T(k): k = 0,..., N\}$ and $\{\Delta_T(k): k = 0,..., N\}$ for the parameters of the modulator and $\{G_R(k): k = 0,..., N\}$ and $\{\Delta_R(k): k = 0,..., N\}$ for the parameters of the demodulator. Now, rewrite (21), (22), (23) and (24) as

$$X(k) = G_T^*(k)\widetilde{X}(k) + \Delta_T^*(k)\widetilde{X}^*(-k), \qquad (25)$$

$$X(-k) = G_T(k)\widetilde{X}(-k) + \Delta_T(k)\widetilde{X}^*(k), \qquad (26)$$

and

$$\begin{aligned} &X(k) - (\Delta_T(k)/G_T(k))^* \cdot (X(-k))^* \\ &= G_T^*(k)[1 - (\Delta_T^*(k)/G_T^*(k))^2]\widetilde{X}(k) \end{aligned}, \qquad (27)$$

$$\begin{aligned} &X(-k) - (\Delta_T(k)/G_T(k)) \cdot (X(k))^* \\ &= G_T(k)[1 - (\Delta_T(k)/G_T(k))^2]\widetilde{X}(-k) \end{aligned}, \qquad (28)$$

[0045] Suppose the demodulator is an ideal demodulator, i.e., the I and Q channels are perfectly matched, so that $\hat{X}(k) = C(k)X(k)$ and $\hat{X}(-k) = C(-k)X(-k)$ at the output of the bank of sub-carrier demodulator 250 (Figure 2), where the complex numbers *C(k)* and *C(-k)* are the channel responses to the sub-carriers $\omega + \omega_k$ and $\omega - \omega_k$, respectively, which may be different if the transmission path includes some radio and intermediate frequency

stages such as mixers, band-pass filters, and IF amplifiers. For local training or calibration shown in Figure 1, $C(k)=C(-k)=c_k$. This demodulator is used to demodulate the signal sent by the balancing modulator unit 165 to be balanced. This training procedure may be carried out as follows:

[0046]   Step A: let $\{\widetilde{X}(k), \widetilde{X}(-k)\} = \{a + j0, \ 0 + j0\}$ at the input of the modulator, which outputs $\mathrm{Re}[Y(t) \cdot \exp(j\omega \cdot t)]$ with

$Y(t) = X(k) \cdot \exp(j2\pi k\Delta_F t) + X(-k) \cdot \exp(-j2\pi k\Delta_F t)$. The corresponding output of the ideal demodulator will be $X1_k = X(k) = c_k G_T^*(k)$ and $X1_{-k} = X(-k) = c_k^* \Delta_T(k)$.

[0047]   Step B: Next, let $\{\widetilde{X}(k), \widetilde{X}(-k)\} = \{0 + j0, a + j0\}$ at the modulator input. The corresponding outputs of the demodulator will be $X2_k = X(k) = c_k^* \Delta_T^*(k)$ and $X2_{-k} = X(-k) = c_k G_T(k)$.

[0048]   Note that (0+j0) and (a+j0) are null and non-null complex numbers. The constant a may be any suitable number. In one embodiment, the constant a is unity ("1"). Furthermore, the non-null complex number can be any constant complex number since it does not change the ratio of $X1_{-k} / X1_k^*$ and $X2_k / X2_{-k}^*$.

[0049]   Combining the demodulation results from the above two steps, it can be shown that $X1_{-k} / X1_k^* = \Delta_T(k) / G_T(k)$ and $X2_k / X2_{-k}^* = \Delta_T^*(k) / G_T^*(k)$. The I-Q balancing blocks are shown in Figure 5. The I-Q balancing blocks 315 ensure that the resulted modulated signal has no "cross-talks" between $X(k)$ and $X(-k)$ but there is a difference from the desired $\{X(k), X(-k)\}$ up to some constant complex scales which can be easily removed after further calibration or training.

[0050]   Consider the situation when the demodulator is non-ideal or is not I-Q balanced. The same channel (I or Q) of the demodulator can be used to receive the signal from the modulator, assuming that the modulator and demodulator use I-Q reference signals ($\cos(\omega \ t)$ and $-\sin(\omega \ t)$) derived from the same reference oscillator. For

example, I-channel of the demodulator is always used for training the modulator. The modulator will repeatedly transmit the same training signal twice for the training signal pair $\{\widetilde{X}(k), \widetilde{X}(-k)\} = \{a + j0, 0 + j0\}$ while the reference signal of the I mixer will be $\cos(\omega t)$ during the first time training and $-\sin(\omega t)$ during the second time training. The results are that the I-channel output is the in-phase component of the transmitted signal in the first time training and the quadrature component in the second time training. Use buffer to store the samples of the I-channel output and then form complex samples by combining the two sets of samples in the buffer and post-process them to obtain desired parameters. Similar procedure can be performed for the training signal pair $\{\widetilde{X}(k), \widetilde{X}(-k)\} = \{0 + j0, a + j0\}$.

[0051]   Consider how to obtain balancing parameters for a I-Q mis-matched demodulator. Assuming that a I-Q balanced modulator is available. This modulator is used to generate modulated signals at the input of the demodulator to be I-Q balanced. At the outputs of the bank of sub-carrier demodulators 250 (Figure 2):

$$\hat{X}(k) = G_R^*(k)C(k)X(k) + \Delta_R^*(k)(C(-k)X(-k))^*, \tag{29}$$

$$\hat{X}(-k) = G_R(k)C(-k)X(-k) + \Delta_R(k)(C(k)X(k))^*, \tag{30}$$

where the complex numbers $C(k)$ and $C(-k)$ are the channel responses to the sub-carriers $\omega + \omega_k$ and $\omega - \omega_k$, respectively, which may be different (for remote training) if the transmission path includes some radio and intermediate frequency stages such as mixers, band-pass filters, and IF amplifiers.

[0052]   If the modulator sends signals modulated with symbols $\{X(k), X(-k)\} = \{a + j0, \ 0 + j0\}$, then, according to (29) and (30), at the output of the demodulator, there are $\hat{X}1_k = G_R^*(k)C(k)$ and $\hat{X}1_{-k} = \Delta_R(k)(C(k))^*$. Therefore, $\hat{X}1_{-k} / \hat{X}1_k^* = \Delta_R(k) / G_R(k)$.

[0053]   Alternatively, the modulator may send $\{X(k), X(-k)\} = \{0 + j0, a + j0\}$ and the outputs of the demodulator are $\hat{X}2_k = \Delta_R^*(k)(C(-k))^*$ and $\hat{X}2_{-k} = G_R(k)C(-k)$. Now we have $\hat{X}2_k / \hat{X}2_{-k}^* = \Delta_R^*(k) / G_R^*(k)$.

[0054]   Once $\Delta_R(k) / G_R(k)$ or $\Delta_R^*(k) / G_R^*(k)$ is determined, the I-Q balancing block can be obtained up to some complex constants according to the following equations

$$\begin{aligned} &\hat{X}(k) - (\Delta_R(k)/G_R(k))^* \cdot (\hat{X}(-k))^* \\ &= G_R^*(k)[1 - (\Delta_R^*(k)/G_R^*(k))^2]X(k) \end{aligned} \tag{31}$$

$$\begin{aligned} &\hat{X}(-k) - (\Delta_R(k)/G_R(k)) \cdot (\hat{X}(k))^* \\ &= G_R(k)[1 - (\Delta_R(k)/G_R(k))^2]X(-k) \end{aligned} \tag{32}$$

[0055]   Another approach to balancing a demodulator is to use some training signal sent remotely by an I-Q balanced modulator.

[0056]   <u>Non-orthogonality at the I and Q mixers:</u>

[0057]   The reference signals, $\cos(\omega t - \varphi)$ and $-\sin(\omega t + \varphi)$, for the I and Q mixers may not be orthogonal (i.e., the phase difference is not be 90 degrees if $\varphi \neq 0$ as shown in Figure 4), which may cause "cross-talks" between the In-phase component and the Quadrature component.

[0058]   Let the base-band signal at the input of the I-Q demodulator be

$$\text{Re}[Y(t) \cdot \exp(j\omega \cdot t)] = \text{Re}[Y(t)]\cos(\omega t) - \text{Im}[Y(t)]\sin(\omega t), \tag{33}$$

where Re[.] is the real part of complex variable and Im[.] is the imaginary part of complex variable. Then at the I and Q mixer outputs, the complex representation of the signal is

$$\text{Re}[Y(t)]\cos\varphi - \text{Im}[Y(t)]\sin\varphi + j(\text{Im}[Y(t)]\cos\varphi - \text{Re}[Y(t)]\sin\varphi), \tag{34}$$

where the terms of frequency higher than the carrier frequency are omitted.

[0059]   Define an operator $\Im_{G,\theta}$ such that $\Im_{G,\theta}(.)$ means multiplying G and introducing phase offset θ to the argument. Let G_I and θ_I be the profile of the I-Channel and let G_Q and θ_Q be profile of the Q-channel, then in Figure 2 ,

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$$\hat{Y}(t) = \mathfrak{I}_{G_I,\theta_I}(\text{Re}[Y(t)]\cos\varphi - \text{Im}[Y(t)]\sin\varphi)$$

$$+ j\mathfrak{I}_{G_Q,\theta_Q}((\text{Im}[Y(t)]\cos\varphi - \text{Re}[Y(t)]\sin\varphi)) =$$

$$= \cos\varphi \cdot \{\mathfrak{I}_{G_I,\theta_I}(\text{Re}[Y(t)]) + j\mathfrak{I}_{G_Q,\theta_Q}(\text{Im}[Y(t)])\}$$

$$- j\sin\varphi \cdot \{\mathfrak{I}_{G_Q,\theta_Q}(\text{Re}[Y(t)]) - j\mathfrak{I}_{G_I,\theta_I}(\text{Im}[Y(t)])\} = \qquad (35)$$

$$= \cos\varphi \cdot \{\mathfrak{I}_{G_I,\theta_I}(\text{Re}[Y(t)]) + j\mathfrak{I}_{G_Q,\theta_Q}(\text{Im}[Y(t)])\}$$

$$- j\sin\varphi \cdot \{\mathfrak{I}_{G_Q,\theta_Q}(\text{Re}[Y(t)]) + j\mathfrak{I}_{-G_I,\theta_I}(\text{Im}[Y(t)])\} =$$

$$= \cos\varphi \cdot \hat{Y}_1(t) - j\sin\varphi \cdot \hat{Y}_2(t),$$

where in the second braces $-\mathfrak{I}_{G_I,\theta_I} = \mathfrak{I}_{-G_I,\theta_I}$ is used. The results derived previously can be applied to the terms in the first braces and second braces individually, with different gain and group delay profiles which will result in corresponding balancing parameters.

[0060]   Consider the situation with two sub-carriers in the above formula and $Y(t) = X(k) \cdot \exp(j2\pi k\Delta_F t) + X(-k) \cdot \exp(-j2\pi k\Delta_F t)$ for some $k>0$. Note that now $G_I = G_I(k)$, $G_Q = G_Q(k)$, $\theta_I = \theta_I(k)$ and $\theta_Q = \theta_Q(k)$ at the frequency $k\Delta_F$. Apply the results derive above to $\hat{Y}_1(t)$ and $\hat{Y}_2(t)$ and let the "balancing parameters" for $\hat{Y}_1(t)$ and $\hat{Y}_2(t)$ be $\{G_1(k), \Delta_1(k)\}$ and $\{G_2(k), \Delta_2(k)\}$, respectively. For example,

$$G_1(k) = G_c(k) + jG_s(k) \text{ and } \Delta_1(k) = (\Delta_c(k) + j\Delta_s(k)) \qquad (36)$$

with

$$G_c(k) = (G_I(k)\cos\theta_I(k) + G_Q(k)\cos\theta_Q(k))/2, \qquad (37)$$

$$G_s(k) = (G_I(k)\sin\theta_I(k) + G_Q(k)\sin\theta_Q(k))/2, \qquad (38)$$

$$\Delta_c(k) = (G_I(k)\cos\theta_I(k) - G_Q(k)\cos\theta_Q(k))/2, \qquad (39)$$

and

$$\Delta_s(k) = (G_I(k)\sin\theta_I(k) - G_Q(k)\sin\theta_Q(k))/2, \qquad (40)$$

etc.

[0061]   Then, let $\omega_k = 2\pi k\Delta_F$,

$$\hat{X}(k) = \frac{1}{T}\int_0^T \hat{Y}(t)(\cos\omega_k t - j\sin\omega_k t)dt =$$

$$= \frac{1}{T}\int_0^T (\cos\varphi \cdot \hat{Y}_1(t) - j\sin\varphi \cdot \hat{Y}_2(t))(\cos\omega_k t - j\sin\omega_k t)dt =$$

$$= \cos\varphi \cdot \hat{X}_1(k) - j\sin\varphi \cdot \hat{X}_2(k) =$$

$$= \cos\varphi \cdot \{G_1^*(k)X(k) + \Delta_1^*(k)X^*(-k)\} \qquad (41)$$

$$- j\sin\varphi \cdot \{G_2^*(k)X(k) + \Delta_2^*(k)X^*(-k)\} =$$

$$= (\cos\varphi \cdot G_1^*(k) - j\sin\varphi \cdot G_2^*(k)) \cdot X(k)$$

$$+ (\cos\varphi \cdot \Delta_1^*(k) - j\sin\varphi \cdot \Delta_2^*(k)) \cdot X^*(-k),$$

similarly,

$$\hat{X}(-k) = \frac{1}{T}\int_0^T \hat{Y}(t)(\cos\omega_k t + j\sin\omega_k t)dt =$$

$$= \cos\varphi \cdot \hat{X}_1(-k) - j\sin\varphi \cdot \hat{X}_2(-k) =$$

$$= \cos\varphi \cdot \{G_1(k)X(-k) + \Delta_1(k)X^*(k)\} \qquad (42)$$

$$- j\sin\varphi \cdot \{G_2(k)X(-k) + \Delta_2(k)X^*(k)\} =$$

$$= (\cos\varphi \cdot G_1(k) - j\sin\varphi \cdot G_2(k)) \cdot X(-k)$$

$$+ (\cos\varphi \cdot \Delta_1(k) - j\sin\varphi \cdot \Delta_2(k)) \cdot X^*(k)$$

[0062] In general, for any $k>0$, there is a pair

$$\hat{X}(k) = (G_{1c}^*(k) - jG_{2s}^*(k)) \cdot X(k) + (\Delta_{1c}^*(k) - j\Delta_{2s}^*(k)) \cdot X^*(-k), \qquad (43)$$

$$\hat{X}(-k) = (G_{1c}(k) - jG_{2s}(k)) \cdot X(-k) + (\Delta_{1c}(k) - j\Delta_{2s}(k)) \cdot X^*(k) \qquad (44)$$

where $G_{1c}(k) = \cos\varphi \cdot G_1(k)$, $G_{2s}(k) = \sin\varphi \cdot G_2(k)$, $\Delta_{1c}(k) = \cos\varphi \cdot \Delta_1(k)$ and

$\Delta_{2s}(k) = \sin\varphi \cdot \Delta_2(k)$.

[0063] Solving the equations (43) and (44) for *X(k)* and *X(-k)*:

$$\hat{X}(k) - \frac{\Delta_{1c}^{*}(k) - j\Delta_{2s}^{*}(k)}{G_{1c}^{*}(k) + jG_{2s}^{*}(k)} \cdot \hat{X}^{*}(-k) =$$

$$= \{(G_{1c}^{*}(k) - jG_{2s}^{*}(k)) - \frac{(\Delta_{1c}^{*}(k) - j\Delta_{2s}^{*}(k))(\Delta_{1c}^{*}(k) + j\Delta_{2s}^{*}(k))}{G_{1c}^{*}(k) + jG_{2s}^{*}(k)}\}X(k) \tag{45}$$

and

$$\hat{X}(-k) - \frac{\Delta_{1c}(k) - j\Delta_{2s}(k)}{G_{1c}(k) + jG_{2s}(k)} \cdot \hat{X}^{*}(k) =$$

$$= \{(G_{1c}(k) - jG_{2s}(k)) - \frac{(\Delta_{1c}(k) - j\Delta_{2s}(k))(\Delta_{1c}(k) + j\Delta_{2s}(k))}{G_{1c}(k) + jG_{2s}(k)}\}X(-k) \tag{46}$$

[0064]   Consider how to obtain balancing parameters for a I-Q mis-matched demodulator. Assuming that a I-Q balanced modulator is available. This modulator is used to generate modulated signals at the input of the demodulator to be I-Q balanced. Based on (43) and (44), at the output of the bank of sub-carrier demodulator:

$$\hat{X}(k) = (G_{1c}^{*}(k) - jG_{2s}^{*}(k)) \cdot C(k)X(k) + (\Delta_{1c}^{*}(k) - j\Delta_{2s}^{*}(k)) \cdot C^{*}(-k)X^{*}(-k), \tag{47}$$

$$\hat{X}(-k) = (G_{1c}(k) - jG_{2s}(k)) \cdot C(-k)X(-k) + (\Delta_{1c}(k) - j\Delta_{2s}(k)) \cdot C^{*}(k)X^{*}(k) \tag{48}$$

where the complex numbers $C(k)$ and $C(-k)$ are the channel responses to the sub-carriers $\omega + \omega_k$ and $\omega - \omega_k$, respectively, which may be different if the transmission path includes some radio and intermediate frequency stages such as mixers, band-pass filters, and IF amplifiers.

[0065]   If the modulator sends signal modulated with symbols $\{X(k), X(-k)\} = \{a + j0, 0 + j0\}$, then, according to (47) and (48), at the output of the demodulator, there are two complex numbers $\hat{X}(k) = X1_k$ and $\hat{X}(-k) = X1_{-k}$, and

$$X1_k = (G_{1c}^{*}(k) - jG_{2s}^{*}(k)) \cdot C(k) \text{ and}$$

$$X1_{-k} = (\Delta_{1c}(k) - j\Delta_{2s}(k)) \cdot C^{*}(k). \tag{49}$$

[0066]   Therefore, $\dfrac{X1_{-k}}{X1_k^{*}} = \dfrac{X1_{-k}X1_k}{|X1_k|^2} = \dfrac{\Delta_{1c}(k) - j\Delta_{2s}(k)}{G_{1c}(k) + jG_{2s}(k)}$ \hfill (50)

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[0067]   On the other hand, the modulator may send $\{X(k), X(-k)\} = \{0 + j0, a + j0\}$ and the outputs of the demodulator are $\hat{X}(k) = X2_k$ and $\hat{X}(-k) = X2_{-k}$, there are

$$X2_k = (\Delta_{1c}^*(k) - j\Delta_{2s}^*(k)) \cdot C^*(-k) \text{ and } X2_{-k} = (G_{1c}(k) - jG_{2s}(k)) \cdot C(-k).$$

[0068]   Therefore, $\dfrac{X2_k}{X2_{-k}^*} = \dfrac{X2_k X2_{-k}}{|X2_{-k}|^2} = \dfrac{\Delta_{1c}^*(k) - j\Delta_{2s}^*(k)}{G_{1c}^*(k) + jG_{2s}^*(k)}$ (51)

[0069]   Once $\Delta G(-k) = \dfrac{\Delta_{1c}^*(k) - j\Delta_{2s}^*(k)}{G_{1c}^*(k) + jG_{2s}^*(k)}$ and $\Delta G(k) = \dfrac{\Delta_{1c}(k) - j\Delta_{2s}(k)}{G_{1c}(k) + jG_{2s}(k)}$ are

obtained, we have the I-Q balancing basic block for the demodulator shown in Figure 5, where the constant complex scales for the output of the basic balancing block are

$$f(\Delta, G, k) = \{(G_{1c}^*(k) - jG_{2s}^*(k)) - \frac{(\Delta_{1c}^*(k) - j\Delta_{2s}^*(k))(\Delta_{1c}^*(k) + j\Delta_{2s}^*(k))}{G_{1c}^*(k) + jG_{2s}^*(k)}\} \text{ and}$$

$$f(\Delta, G, -k) = \{(G_{1c}(k) - jG_{2s}(k)) - \frac{(\Delta_{1c}(k) - j\Delta_{2s}(k))(\Delta_{1c}(k) + j\Delta_{2s}(k))}{G_{1c}(k) + jG_{2s}(k)}\} \text{ The}$$

balancing parameters are usually fixed during normal communication processes. At the output of the balancing block, the constant complex scales can be removed by demodulating the pilot symbol or preamble symbol sent by the remote party during the communication process.

[0070]   Another approach to balancing a demodulator is to use some training signal sent remotely by an I-Q balanced modulator of a remote party.

[0071]   For modulators, similar results can be obtained as discussed above. Suppose there is a modulator described in Figure 3 with I-Q imbalance such as reference phase offset, gain and group imbalance. In order to generate the desired modulating signal at the output of the modulator with modulating symbols $\{X(k): k=-N, ..., -k, ...k, ..., N\}$, $\widetilde{X}(k)$ should be applied to the input of the modulator by introducing "cross-talks" between $X(k)$ and $X(-k)$. According to equation (43) and (44), there are

$$X(k) = (G_{1c}^*(k) - jG_{2s}^*(k)) \cdot \widetilde{X}(k) + (\Delta_{1c}^*(k) - j\Delta_{2s}^*(k)) \cdot \widetilde{X}^*(-k),   (52)$$

$$X(-k) = (G_{1c}(k) - jG_{2s}(k)) \cdot \widetilde{X}(-k) + (\Delta_{1c}(k) - j\Delta_{2s}(k)) \cdot \widetilde{X}^*(k) \tag{53}$$

[0072]   Solving equations (5) and (6), we have,

$$X(k) - \frac{\Delta_{1c}^*(k) - j\Delta_{2s}^*(k)}{G_{1c}^*(k) + jG_{2s}^*(k)} \cdot X^*(-k) =$$

$$= \{(G_{1c}^*(k) - jG_{2s}^*(k)) - \frac{(\Delta_{1c}^*(k) - j\Delta_{2s}^*(k))(\Delta_{1c}^*(k) + j\Delta_{2s}^*(k))}{G_{1c}^*(k) + jG_{2s}^*(k)}\} \widetilde{X}(k) \tag{54}$$

and

$$X(-k) - \frac{\Delta_{1c}(k) - j\Delta_{2s}(k)}{G_{1c}(k) + jG_{2s}(k)} \cdot X^*(k) =$$

$$= \{(G_{1c}(k) - jG_{2s}(k)) - \frac{(\Delta_{1c}(k) - j\Delta_{2s}(k))(\Delta_{1c}(k) + j\Delta_{2s}(k))}{G_{1c}(k) + jG_{2s}(k)}\} \widetilde{X}(-k) \tag{55}$$

[0073]   The resulting basic balancing block for the demodulator and the modulator is shown in Figure 5.

[0074]   Using approaches similar to those in the above, the balancing above parameters can be obtained for the modulator. After appropriately scaled, $\widetilde{X}(k)$ and $\widetilde{X}(-k)$ can be obtained for the corresponding sub-carrier modulators.

[0075]   Figure 5 is a diagram illustrating the basic balancing block 265 shown in Figure 2 according to one embodiment of the invention. The basic block 265 includes first and second balancer 510 and 530, first and second balancing parameters 520 and 540, and first and second adders/subtractors 550 and 560. The input signals to the basic block 265 are signals U(k) and U(-k) indexed symmetrically to 0. For modulators, the signals U(k) and U(-k) are the desired modulating signals modulating a pair of sub-carrier modulators whose carrier frequencies are symmetrical to the center frequency of the resulting multi-carrier composite signal. For demodulators, the signals U(k) and U(-k) are outputs of a pair of two sub-carrier demodulators whose frequencies are symmetrical to the center frequency of the multi-carrier composite signal. The output signals from the basic block 265 are W(k) and W(-k). For modulators, the signals W(k) and W(-k) are pre-

distorted or pre-compensated and fed into a pair of sub-carrier modulators whose frequencies are symmetrical to the center frequency of the multi-carrier composite signal, and after properly scaled, resulting in the effects that U(k) and U(-k) are modulating the pair from view point of any remote receivers. For demodulators, the signals W(k) and W(-k) are, up to some constant complex scales, recovered original signals that modulate a pair of sub-carrier modulators of the multi-carrier composite signal from remote transmitters.

[0076]   The first balancer 510 generates a first balancing signal from U(k) of index k corresponding to the k-th sub-carrier modulator/demodulator at the sub-carrier frequency $k\Delta_F$. The first subtractor 550 subtracts the first balancing signal from U(-k) of index –k. The two indices of the related signals are symmetrical with respect to index 0 which corresponds to a center frequency of the final composite multi-carrier signal. The first subtractor 550 generating a first balanced signal W(-k) of index –k corresponding to the sub-carrier frequency $-k\Delta_F$. The first balanced signal W(-k) is a first desired signal scaled by a first complex factor.

[0077]   The first balancer 510 includes a first converter 512 and a first multiplier 515. The first converter 512 converts the first signal U(k) into a first complex conjugate U*(k). The first multiplier 515 multiplies the first complex conjugate U*(k) with the first balancing parameter 520 to generate the first balancing signal V(k). The first balancing parameter 520 is obtained by a training sequence as described above.

[0078]   The second balancer 530 generates a second balancing signal from U(-k) of index -k. The second subtractor 560 subtracts the second balancing signal from U(k) of index k. The two indices of related signals are symmetrical with respect to index 0 that corresponds to a center frequency of the composite multi-carrier signal. The second subtractor 560 generates a second balanced signal W(k) corresponding to the sub-carrier frequency $k\Delta_F$. The second balanced signal W(k) is a second desired signal scaled by a second complex factor.

[0079]   The second balancer 530 includes a second converter 532 and a second multiplier 535. The second converter 512 converts the second signal U(-k) into a second

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complex conjugate U*(-k). The second multiplier 535 multiplies the second complex conjugate U*(-k) with the second balancing parameter 540 to generate the second balancing signal V(-k). The second balancing parameter 540 is obtained by a training sequence as described above.

[0080]   Now let's consider how to remove the I-Q imbalance effects on the 0-th sub-carrier signal for demodulators (i.e., for $k=0$, the sub-carrier at the center frequency of the radio multi-carrier composite signal or at the DC frequency of the baseband multi-carrier signal). According to equations (9) and (41) with $k=0$ or (42) with $k=0$, the output of the 0-th sub-carrier demodulator is a linear combination of the original modulating signal and its conjugate. Therefore, without loss of generality, we can assume

$$\hat{X}(0) = G^*(0)X(0) + \Delta(0)X^*(0) \tag{56}$$

where $G^*(0)$ and $\Delta(0)$ are two complex numbers depending on the I-Q imbalance conditions of the I-Q demodulator at DC.

[0081]   Solving equation (56), we obtain

$$G(0)\hat{X}(0) - \Delta(0)\hat{X}^*(0) = (|G(0)|^2 - |\Delta(0)|^2)X(0) \tag{57}$$

$$\text{or} \quad \hat{X}(0) - \frac{\Delta(0)}{G(0)}\hat{X}^*(0) = G^*(0)(1 - \left|\frac{\Delta(0)}{G(0)}\right|^2)X(0) \tag{58}$$

[0082]   In this case, two identified inputs are applied to the basic balancing block in Figure 5 and only one output is needed. Therefore only one branch in Figure 5 is needed to generate the resulting balanced signal with one balancing signal and one balancing parameter and the other branch is redundant.

[0083]   To obtain the balancing parameter for the 0-th sub-carrier signal, the training signal source needs to send two training signals for the 0-th sub-carrier demodulator. The first training signal contains the 0-th sub-carrier signal modulated by $X(0) = C_0$ where C_0 is any given non-null complex number, and, according to equation (56), the output of the 0-th sub-carrier demodulator is

$$X1 = G^*(0)C_0 + \Delta(0)C_0^* \tag{59}$$

[0084]   The second training signal contains the 0-th sub-carrier signal modulated by $X(0) = jC_0$ (i.e., 90 degree phase shift of the first modulating signal), and, according to equation (56), the output of the 0-th sub-carrier demodulator is

$$X2 = G^*(0)jC_0 - \Delta(0)jC_0^* \tag{60}$$

[0085]   Combining equations (59) and (60), we obtain

$$X1 - jX2 = 2 \cdot G^*(0)C_0 \quad \text{and} \quad X1 + jX2 = 2 \cdot \Delta(0)C_0^*.$$

[0086]   Therefore,

$$\frac{X1 + jX2}{(X1 - jX2)^*} = \frac{X1 + jX2}{\left(X1^* + jX2^*\right)} = \frac{\Delta(0)}{G(0)} \tag{61}$$

[0087]   Similar results can be obtained for modulator. There are

$$X(0) = G^*(0)\widetilde{X}(0) + \Delta(0)\widetilde{X}^*(0),$$

$$G(0)X(0) - \Delta(0)X^*(0) = (|G(0)|^2 - |\Delta(0)|^2)\widetilde{X}(0)$$

and $\widetilde{X}(0) - \dfrac{\Delta(0)}{G(0)}\widetilde{X}^*(0) = G(0)(1 - \left|\dfrac{\Delta(0)}{G(0)}\right|^2)X(0)$. The balancing parameter can be

obtained in a similar way to the one used for demodulators.

[0088]   While this invention has been described with reference to illustrative embodiments, this description is not intended to be construed in a limiting sense. Various modifications of the illustrative embodiments, as well as other embodiments of the invention, which are apparent to persons skilled in the art to which the invention pertains are deemed to lie within the spirit and scope of the invention.